Research Statement

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My research interest is in automated proving of theorems in combinatorics. Or at least using computers to do their best assisting humans to solve the problems. This includes: making conjectures, giving numerical evidence, and doing symbolic computations. I like many areas of combinatorics, my favorites being Combinatorial Game theory, Ramsey theory, and the theory of partitions.

1 Dissertation Research

Richard Hamming, a great applied mathematician of the last century, said "the purpose of computing is insight, not numbers". Understanding makes me feel good. Computation gives me insight in two different ways. First the *act* of programming makes me understand the problem much better, and at a deeper level, and second, the *output* often leads to further understanding. Once we collect all the information, we see the *big picture* without worrying about the details of calculations. I like to solve challenging problems, and it is always the case that knowing computer programming helps the calculational part goes smoother.

During my graduate years, I learned the philosophy and methodology of using computers in mathematical research, specifically symbolic computation and automated proving, from Prof. Doron Zeilberger, my advisor. In my opinion, it is not very important in what mathematical area I am working on, since the *method* and *experience* gained in doing computer-assisted and computer-generated research in one area are likely to be transferable to other areas.

The "Toads and Frogs" game: I do research in combinatorial game theory. The modern theory was developed by J.Conway, E.Berlekamp, R.Guy, who wrote the classic book *Winning Ways*, that mostly dealt with *partizan* games, and by Aviezri Fraenkel and his many students, who study *impartial* games.

I started my research by investigating "Toads and Frogs". In 1994, Jeff Erickson made five intriguing conjectures [2]. In my paper (joint with my advisor), [9], we proved one of Erickson's conjectures. In two other papers of mine, [6], [7] ,I settled three others, two positively and one negatively. In the process, I proved much more general results. More importantly, we discovered a new technique that allowed computers to conjecture and prove values of many families of positions, completely automatically. We called it the “Finite State Method”. In the past, mathematicians had a hard time proving the values of positions by logical reasoning. Once computers are taught how to do this ”reasoning”, they can do much more, of course. I implemented all this in the computer algebra system Maple. My program can also determine a winning move (if it exists) in each position, by using the recurrence relation for each of the positions.

Problems on Schur Triples: Schur's theorem is one of the “super six” theorems in Ramsey theory mentioned in Ron Graham, Bruce Rothschild, and Joel Spencer's famous monograph. The *Schur number*, $s(r)$, is the smallest number such that for any r-coloring of the set $1, 2, ..., s(r)$, there must be a monochromatic solution to the equation $x + y = z$. A triple (x, y, z) is called a *Schur triple*. Only the first four values are currently known: $s(1) = 2, s(2) = 5, s(3) = 14, s(4) = 45$. Even $s(5)$ is unknown. It is a natural question, asked by Ron Graham, what is the least number of Schur triples in $\{1, ..., n\}$ in a 2-coloring, as $n \to \infty$. This problem was solved independently by [4], [5], who shared the 100$ prize offered by Graham. Yet another proof appeared later [1]. The answer is $\frac{n^2}{22} + O(n)$.

Ron Graham further asked about the minimum number of monochromatic triples $(x, y, x + ay)$ in 2-colorings of $[1, n]$. I wrote a Maple program to find the optimal coloring when n is small. After finding the pattern, I developed a method that I called “greedy calculus”, to calculate “good” colorings which gives upper bounds. Then I used another method involving calculus that gives lower bounds. So far the upper bounds and lower bounds do not match. But these upper bounds are conjectured to be sharp. I also applied this algorithm to the original Schur triples, but with r-colorings, to get upper bounds which I suspect to be sharp. These considerably improve previous bounds due to A. Robertson. These results are in [8].

Moment Calculus: The expectation functional is a powerful tool to study combinatorial objects, and often gives you quite useful information. To find higher moments, the computation gets complicated and we need computers to do symbolic computation for us. The technique has already been demonstrated in [10], [11]. Once we find high enough moments, they could actually be useful for calculating lower bounds for enumerating combinatorial objects (see [11]). I have calculated the higher moments of the Ramsey graph of K_3 and K_4 on n vertices for the r-coloring of edges of the complete graph, the second moment of Schur Triples with r-coloring on $[1, n]$. The results are on my website.

2 Future Goals

Using computers in mathematical research is getting more and more common, but most of it is done in “interactive” mode. I believe that a more systematic and methodical approach, with extensive programming, can lead to even more.

I want to keep working in the kind of research compatible with my strengths. Experimental mathematics and symbolic computation are the *key words*. I will try and focus on problems where there is a way for computers to take part in formulating conjectures, delivering numerical results for small examples, doing symbolic computation, and whenever possible, proving interesting new results.

I) Finite State Method For Combinatorial Games:

The work of mine and my advisor shows that the Finite State Method can prove values of positions of large (infinite) families of game-positions, even for games for which computing the value for *arbitrary* positions is NP-hard (e.g. "Toads and Frogs"). I adapted this method for chess endgame problems. I believe that there are still many more games where this method can be used. In general, the values of positions of most combinatorial games seem pattern-less. However there is still the kind of games that the method can solve. I want to adapt this method to the chess endgame problem. The chess endgame problem on an m by n board when White has a Rook and a King and Black only has its King. Given a starting position with a fixed number m, what is the shortest number of moves for white to deliver a checkmate (for (general!) n). .

II) Combinatorial Game Theory:

Ultimate periodicity in Octal games: A Take-And-Break game is the type of impartial game where players take out a specific number of beans according to the rule and split up the pile at the same time. The first player who does not have a move loses. Octal Game is a Take-And-Break game where each game is played according to the assigned octal code digit. In [3], problem 2, Richard Guy asked about the ultimate periodicity of Nim sequence of Octal games. The conjecture is that all the octal games have ultimate periodicity. I already wrote a program to compute the Nim sequence of any octal game.

III) Ramsey Theory:

First problem: Once in awhile, it is fun to try a hand on a very famous problem. Find an asymptotic lower bound of $R(k, k)$. The classic result by Erdos stated $R(k, k) \geq 2^{\frac{k}{2}}$. He use the linear expectation probabilistic method to obtain the lower bound. There is an idea from Doron Zeilberger in [11] about computing higher moment calculus of $R(k, k)$ then applying it to Bonferroni's theorem along with other probabilistic methods like Lovasz local lemma in an effort to try to improve the lower bound. This would be an improvement of the old method does not guarantee a new lower bound. Since this is a very famous problem, the chance of improving the result is definitely slim. At least any new information on this problem would be interesting.

Second problem: As a problem along the same lines as my work on (generalized) Schur triples mentioned above, we can ask a similar problem related to Ramsey numbers instead of Schur numbers. For a fixed number k, find the least number of monochromatic K_k for any edge 2-coloring graph in K_n where $n \to \infty$. The easy solution for K_3 is known. The answer is $\frac{n^3}{24} + O(n^2)$ which is the same as the average. The answer for $k \geq 4$ is unknown.

Third problem: The definition of Schur numbers mentioned in the first section can be generalized to other types of linear equations. These number are called Rado number. For example, find the smallest n so that for any 2-coloring of integers on $[1, n]$ there must be a monochromatic solution to $ax + by = bz$ where a and b are fixed constants. There are many results on many types of equations with 2 coloring. But there is no work done for 3-colorings. It would be a challenge to develop symbolic programming to answer this type of question when $r = 3$.

I am willing to work on anything along these lines, and like to collaborate also.

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